A LOOK AT THE COMMON CORE STATE STANDARDS AND THEIR IMPLICATIONS FOR FORMATIVE ASSESSMENT

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PROFESSIONAL STANDARDS FOR TEACHING MATHEMATICS STATES THE FOLLOWING:

- posing questions and tasks that elicit, engage, and challenge each student's thinking;
- listening carefully to students' ideas;
- asking students to clarify and justify their ideas orally and in writing;
- deciding what to pursue in depth from among the ideas that students bring up during a discussion;
- deciding when and how to attach mathematical notation and language to students' ideas;
- deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty;
- •monitoring students' participation in discussions and deciding when and how to encourage each student to participate. (NCTM 1991)

PROBLEM SOLVING KEY TO COMMON CORE MATH PRACTICES

- Traditional crosswalks of aligning specific standards from state frameworks to common core is dangerous.
- CCSS does not promote a check-off approach.
- Rich problems include many content standards as well as the mathematical practices

WHY USE FORMATIVE ASSESSMENT?

- National Math Panel cited that formative assessment is the single most effective practice in the improvement of student learning.
- Formative assessment is on-going, formal, and informal evaluation of where students are in their learning progressions.
- Formative assessment is often invisible to students

TOOLKIT OF FORMATIVE ASSESSMENT PRACTICES

- Range questions
- Observation protocols
- Gallery walks
- Round-robin activities
- Focused questions/hinged questions
- Mathematical discourse
- Feedback
- Exit cards (tickets to leave)

RESPONSIVE TEACHING

- “I taught it and I know they learned it because I have evidence to support their learning.”
- Requires an adjustment in your lesson plans to meet your students where they are in the learning process.
- Prior knowledge is quickly assessed
  - Range questions
  - Exit cards

FORMATIVE ASSESSMENT:

- Makes student thinking visible
- Range questions
- Exit cards/tickets to leave

GRADE 6 RANGE QUESTION

- Which is greater

$1\ 2/3 \times \frac{3}{4}$ or $1\ 2/3 \div \frac{3}{4}$?

STUDENT A

" I think that they will both be the same because when there is a division sign yoo [sic] have to turn it into a multiplication sign. Both of them have 1 2/3 so you turn the mixed number into an improper fraction. Then they would both be 5/3 x ¾ and 5/3 x ¾."

STUDENT B

" I think that 1 2/3 x ¾ is bigger because times makes number bigger"

STUDENT C

" I think 1 2/3 • ¾ is greater than 1 2/3 ÷ ¾ because 1 2/3 • ¾ is multipling [sic] and 1 2/3 ÷ ¾ is just seeing how many time [sic] ¾ can go into 1 2/3."

EFFECTIVE TEACHING

- Teachers use a coherent and consistent curriculum.
- Teachers design student-centric classrooms
  - Students are actively engaged.
  - Students build on previous knowledge.
- Rich tasks are assigned that:
  - Require persistence and perseverance.
  - Require multiple representations.
  - Require construction of viable arguments.
  - Require students to reason abstractly and quantitatively.
  - Require modeling with mathematics

TEACHER ROLE

- While students are working teachers walk around room listening, recording student dispositions, asking questions
- Teachers must plan to ask “hinge questions” that include engaging, clarifying, and refocusing questions.
- Observation protocols helpful to document student involvement

OBSERVATION PROTOCOLS

	Skill		Learning and Understanding						Student's Habits		
	Opening Activity		Work Time								
Student's Name	Active Learner	Understands question(s)	Shows work (computation diagram, table, graph) equation)	Explains solution	Are the answers accurate?	Group Work	Accountable Talk	Completes work	Help	On Task	Comments

Student Comments
On task
Asks topical questions
Answers questions
Takes notes
Active participant
Copies work from others
Helps others
Accepts help
Accurately applies concepts
Multiple representations
Multiple strategies
Explains process: writing (W), verbal (V), both (B)
Applies previously learned concepts
Correct answer
Computational accuracy
Procedural accuracy
Problem-solving strategies

FOCUSED QUESTIONS

- Engaging
- Clarifying
- Refocusing

ENGAGING QUESTIONS

- Designed to engage students with the problem.
- Well-thought-out engaging questions should spark students' interest and want to make them solve the problem.

TYPICAL ASSIGNMENT

- *Compute: 1,000,000 ÷ 72.*

ENGAGING PROBLEM

- *If you begin counting your heartbeats at exactly twelve o'clock New Year's Day, when will your heart have beaten 1 million times? Support your answer.*

ENGAGING QUESTIONS

- *Do you think each member of your group has the same heart rate?*
- *How can you determine each person's heart rate?*
- *How can you incorporate each person's heart rate in your solution?*
- Suppose students found an average - ask
- *What would happen if you used the median class heart rate?* and
- *How would your results differ?*

CLARIFYING QUESTIONS

- Asked if teacher does not understand student representations, explanations, or the pathway in which students are heading.
- Asked when student work or conversations are muddled.
- Ask students to explain what they are thinking or to clarify the assignment or directions.

SAMPLE CLARIFYING QUESTIONS

- *Are you sure everyone in your group has the same heart rate?*
- *Do you think your heart rate stays the same all day?*
- *How is multiplying your heart rate by 60 seconds in a minute different from dividing your heart rate by 60 seconds in a minute?*
- *Which do you think will best help you begin to determine when your heart rate beats the millionth time?*
- *What were you thinking when you multiplied 1 million by 60 seconds in a minute?*
- *If your heart beats 72 times per minute, how many times will it beat in 2 minutes? 3 minutes? 10 minutes?*

REFOCUSING QUESTIONS

- Prevent students from pursuing a dead-end strategy when solving a particular problem,

- Guide students to answer the question asked, and return them to the task at hand if they drift off.

REFOCUSING QUESTIONS

- *I noticed that you have already started doing calculations that involve multiplication.*
- *Can you explain to me what the calculations represent?*
- *How will they help you find out how many times a heart beats every hour?*

PROBING STUDENT THINKING: INTERVIEWS AND CONFERENCES

- ***Teachers interview individual students to ensure exact understanding of what students are thinking about their problem solving.***

- *A photograph that is 6 inches on the base and 8 inches high is to be enlarged so that the new base is to be 15 inches. What will the height of the enlargement be?*

Student Responses:

- 2.5,
- 17,
- 20, and
- 25 inches

SAMPLE INTERVIEW

- *Teacher:* Can you tell me what you were thinking when you predicted 17 inches as the answer?

- *Student:* That was easy. I saw that 15 is 9 more than 6, so I need a base that is 9 more than 8.

Gallery Walks promote:

- critical thinking;
- written expression;
- oral communication; and
- an interactive, student-centered environment.

Gallery Walk Problems Should Require Students To:

- to analyze,
- predict,
- compare,
- construct, or
- justify

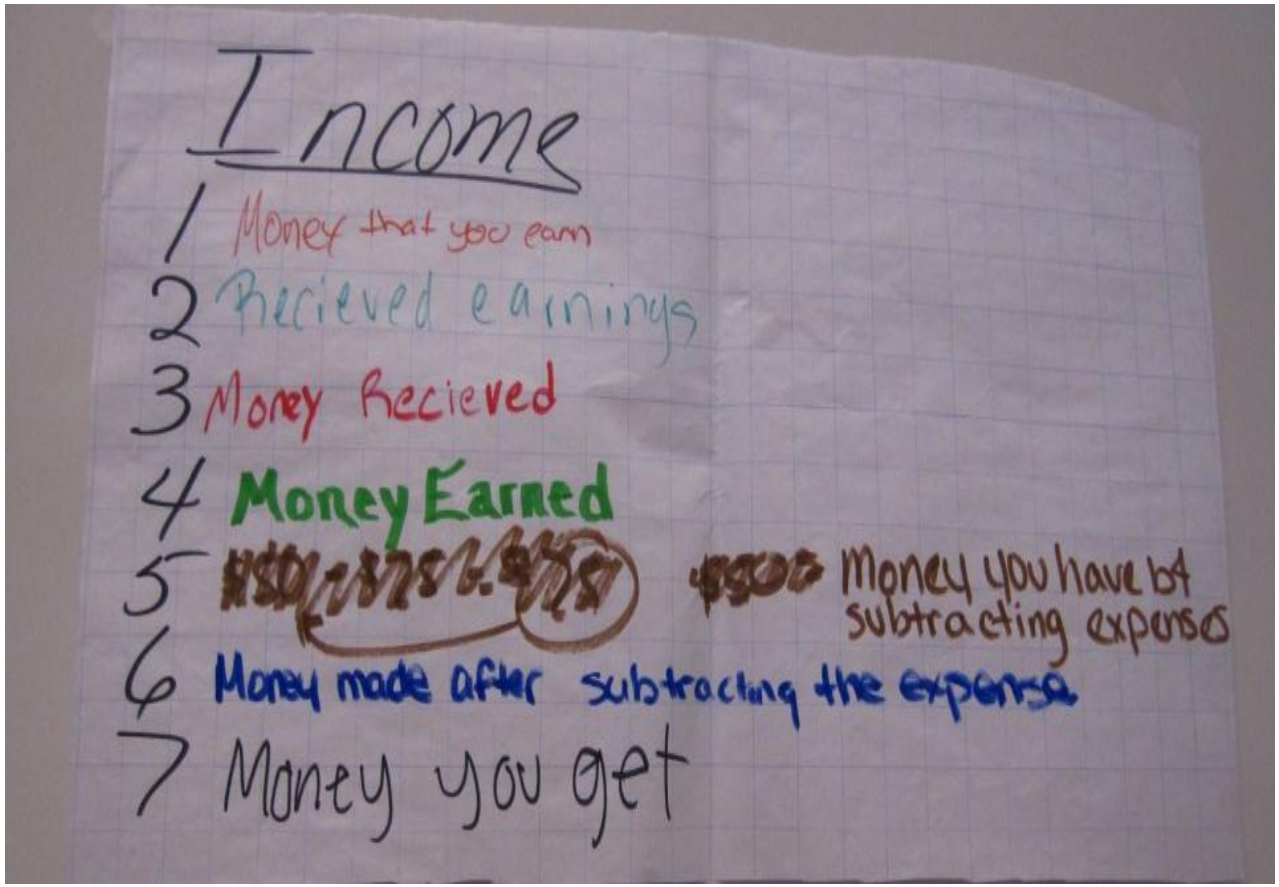
- so that, after they complete the mathematics, a **rich discussion** can follow.

GALLERY WALKS MAY BE USED:

- As formative assessment
- As summative assessment
- To quickly assess what students know and are able to do in less than 10 minutes or so
- Are effective means for assessing math vocabulary

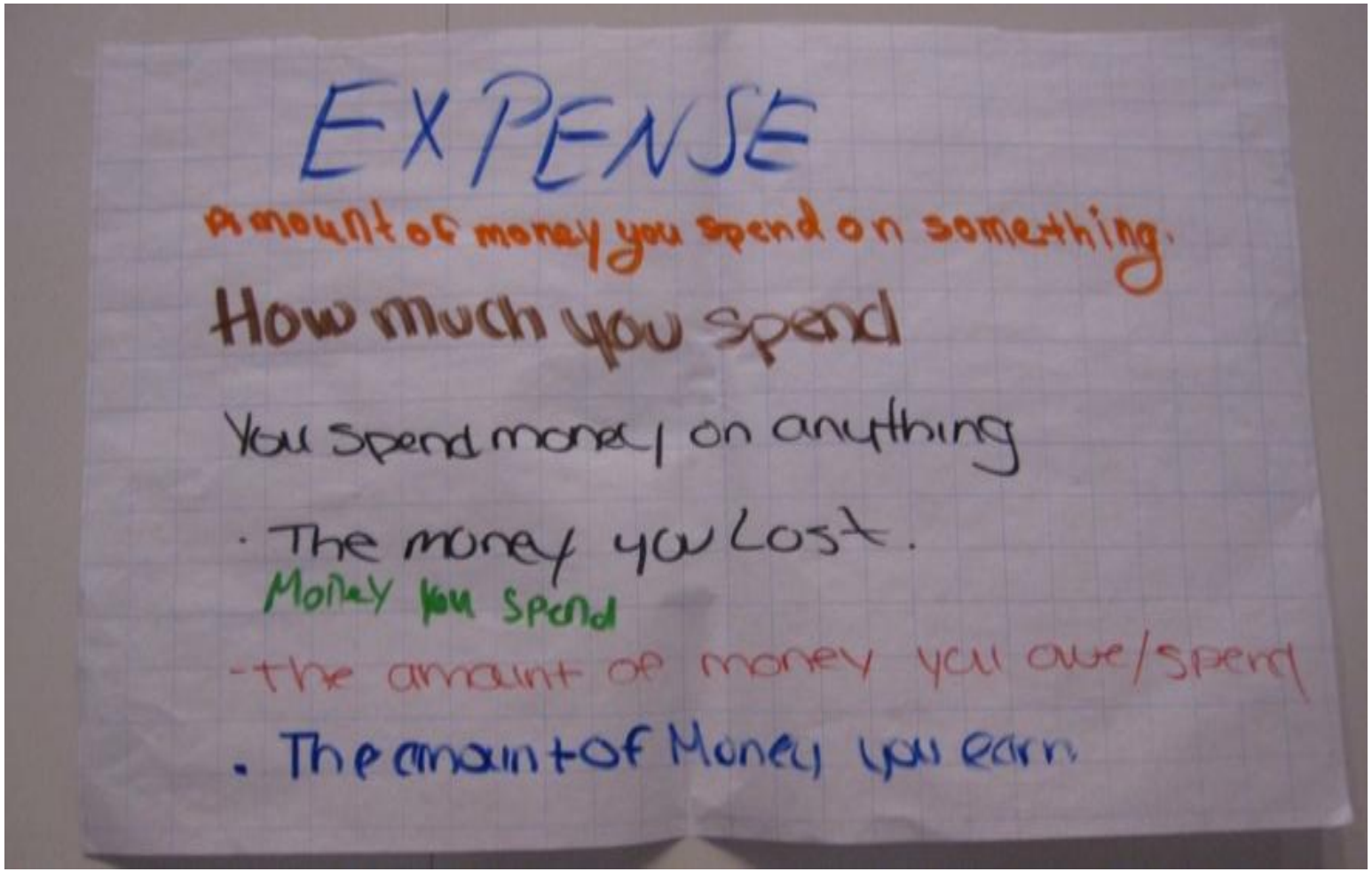
with the progress her students displayed, but she also documented that some misconceptions remained.

INCOME

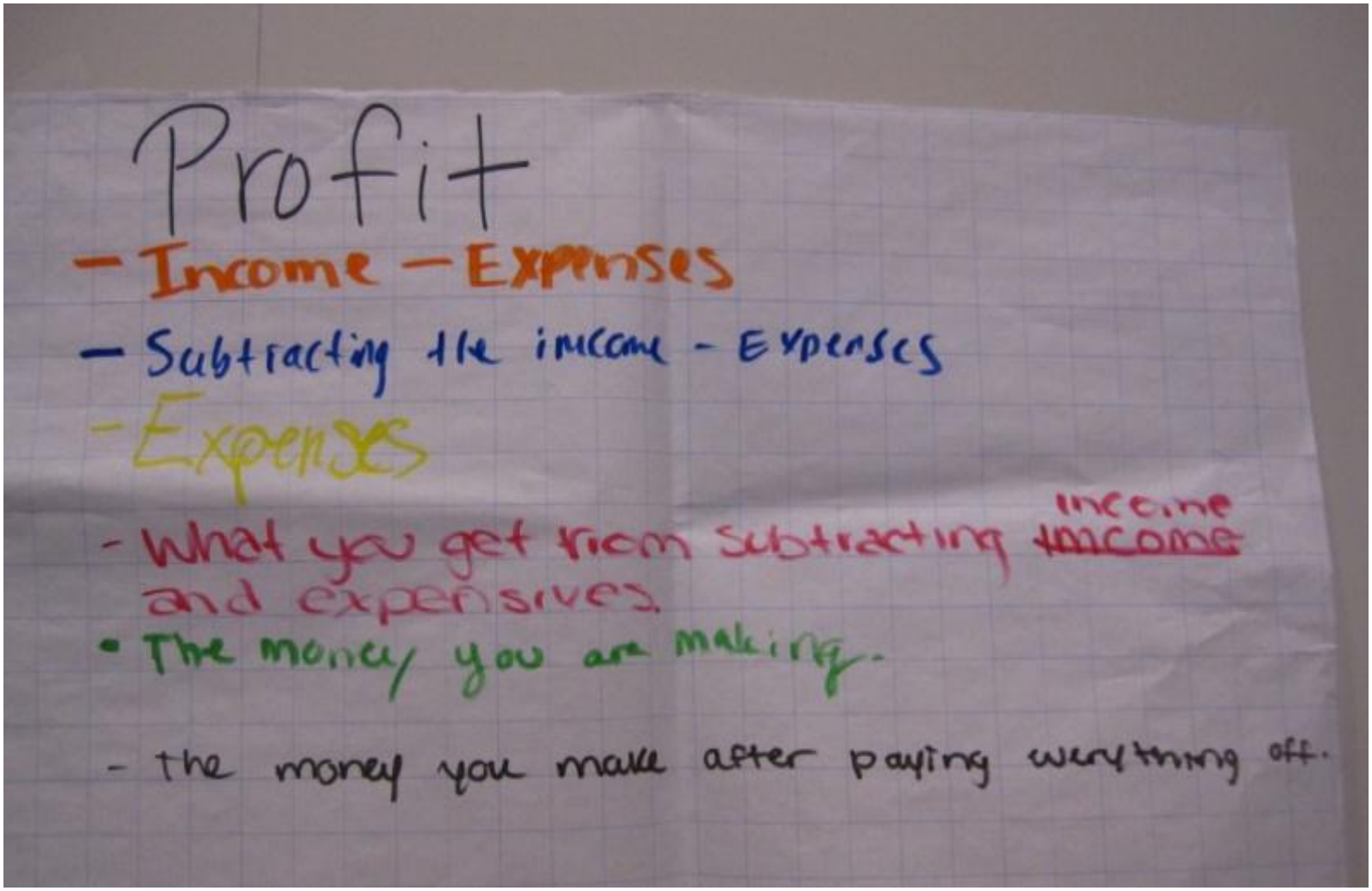


further discussion.

EXPENSE



PROFIT


Profit
- Income - Expenses
- Subtracting the income - Expenses
- Expenses
- What you get from subtracting income and expensives.
• The money you are making.
- the money you make after paying everything off.

COMMON CORE MATHEMATICAL PRACTICES

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

MAKE SENSE OF PROBLEMS AND PERSEVERE IN SOLVING THEM.

- Tasks, problems, activities must be relevant to the students.
- Tasks, problems, activities must have multiple entry points, multiple representations, open-response or open-ended.
- May take many class periods to complete and may have various solution paths.

CONJECTURE BOARDS

- Construct viable arguments and critique the reasoning of others
- Record all student ideas
- Leave conjectures on board until they can be negated or if a counter example can be made
- Promotes deep mathematical thinking and reasoning

ROUND ROBIN ACTIVITIES

- Enables the identification of student misconceptions before they are solidified
- Allows for peer teaching and immediate teacher intervention
- Gets students out of their seats in a constructive manner

THE LOOK OF A ROUND ROBIN

- Students grouped in triads
- Each student assigned a number 1, 2, or 3
- The ones go to the board and write down the expression, equation, etc and does ONE STEP
- The ones sit down and the twos go to the board and does a second step then sits down
- The threes go to the board and does a third step and sits down
- The ones return to board and do another step etc.

MATHEMATICAL MODELING

- Engagement of students in the application of math to problems arising in everyday life.
- Build a tetrahedron with 13 rows and determine the number of individual tetrahedra in each row
- Makes mathematics obvious to students

A TRIP TO THE STATE PARK GRADES 3 - 5

- Make sense of elapsed time.
- Design a schedule.
- Determine cost.
- Design a field trip

GEOMETRY GRADES 7 - 10

- Sports Bag Problem
- Design and make a model of a sports bag
- Understand when to use diameter or circumference as important measure

EMERGENCY RESPONDERS GRADES 8 - 10

- Analyze data to determine which of two ambulance companies is the better choice.
- Construct mathematical arguments to support a position.
- No correct answer…an anomaly for most students.

MATHEMATICAL MODELING IN HIGH SCHOOL

- ‘Mathematical modeling is the link between mathematics and the rest of the world.”

- The process of beginning with a situation and gaining understanding about that situation is generally referred to as “modeling.” If the understanding comes about through the use of mathematics, the process is known as mathematical modeling.

TRIGONOMETRY PROBLEMS

- Student's make a clinometer, go outside and measure the height of the flag pole, a very large tree etc.

- Students determine height of tree/ flag pole in two ways using similar triangles or trig functions

- Real-world activities provide a NEED TO KNOW the math and are more likely to become engaged in recall what they learned.

REASON ABSTRACTLY AND QUANTITATIVELY

- If you had to mentally compute the following, what strategies might you use?

  If 5 chocolates cost $0.75, how much do 13 cost?

MENTALLY COMPUTE

33 X 27

LOOK FOR AND MAKE USE OF STRUCTURE.

- How did the strategies you used link to generalizing arithmetic?

- Did anyone use the difference of two squares?

$(14 + 1)(14 - 1)$

REASON ABSTRACTLY AND QUANTITATIVELY.

- Provide examples of real-world applications to determine whether your students are reasoning quantitatively.

- Encourage mental mathematics

- How much bigger is a trillion than a billion?

- How large a container might you need to hold a million golf balls, dollar bills, etc

HOW MANY ARE THERE?

- How might you determine how many people showed up for the Red Sox World Series parade in 2004?

- How does the media determine the number of folks at any large demonstration?

HIGH SCHOOL CONCEPTUAL CONCEPTS

- The big ideas that connect mathematics across high school
- A progression of increasing complexity
- Description of what or mathematical content to be learned elaborated through domains, clusters and standards

FOCUS ON PROBLEM SOLVING

- US 15-year old students perform at the bottom of all countries taking the PISA which assesses problem solving.

- US students do well on computation but poorly across the grades on problem solving.

HIGH SCHOOL PATHWAYS

- Pathway A: consists of two algebra courses and a geometry course, with some data, probability and statistics infused throughout each (traditional)

- Pathway B: typically seen internationally that consists of a sequence of 3 courses each of which treats aspects of algebra, geometry and data, probability, and statistics.

SOME POSITIVE ASPECTS OF THE COMMON CORE ZAL USISKIN 2011

1. Overall content in the 9-12 standards (but too much to be expected to be learned by all students in 4 years)
2. The development of algebra K-9
3. The definitions of congruence and similarity in terms of geometric transformations
4. The recognition of mathematical modeling as an important mathematical practice
5. The recognition that computer algebra systems (CAS) can play important roles in learning mathematics

NCTM PROCESS STANDARDS AND THE CCSS MATHEMATICAL PRACTICES

NCTM Process Standards	CCSS Standards for Math Practices
◉ Problem Solving	1. Make sense of problems and persevere in solving them. 5. Use appropriate tools strategically
◉ Reasoning &Proof	2. Reason abstractly and quantitatively. 3. Critique the reasoning of others 8. Look for and express regularity in repeated reasoning

- Communication

3. Construct viable arguments

- Connections

6. Attend to precision
7. Look for and make use of structure

- Representations

4. Model with mathematics

INTENDED OUTCOMES OF CCSS

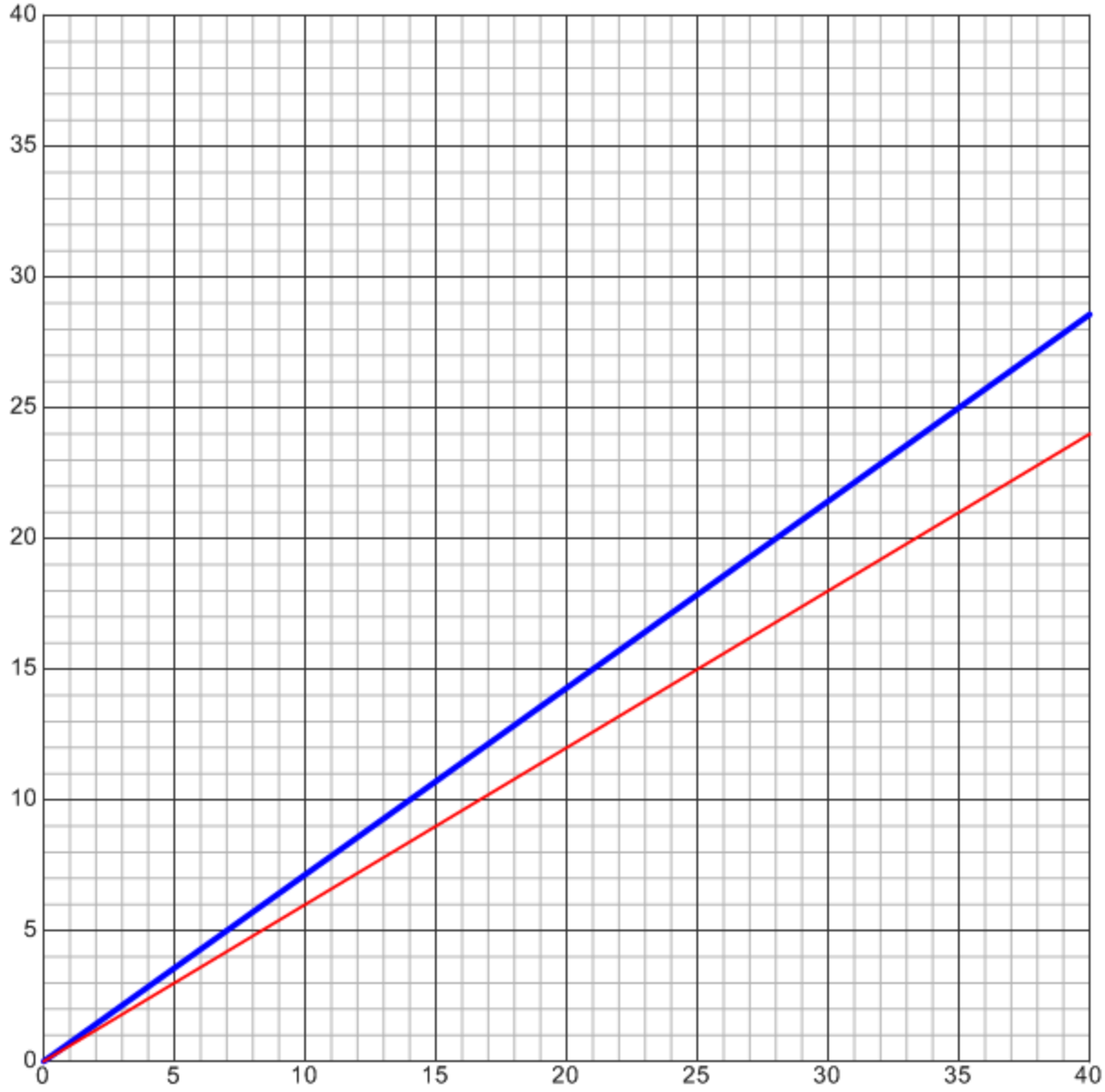
- Teachers change the way in which they engage students in doing meaningful mathematics.

- Students are more actively involved in working with models that make the need to know mathematics visible.

BOTTOM LINE

"If we continue to teach the way in which we always taught (or were taught) we will continue to get the same results and that is not good enough

COMPARE THE RATIOS 5/7 AND3/5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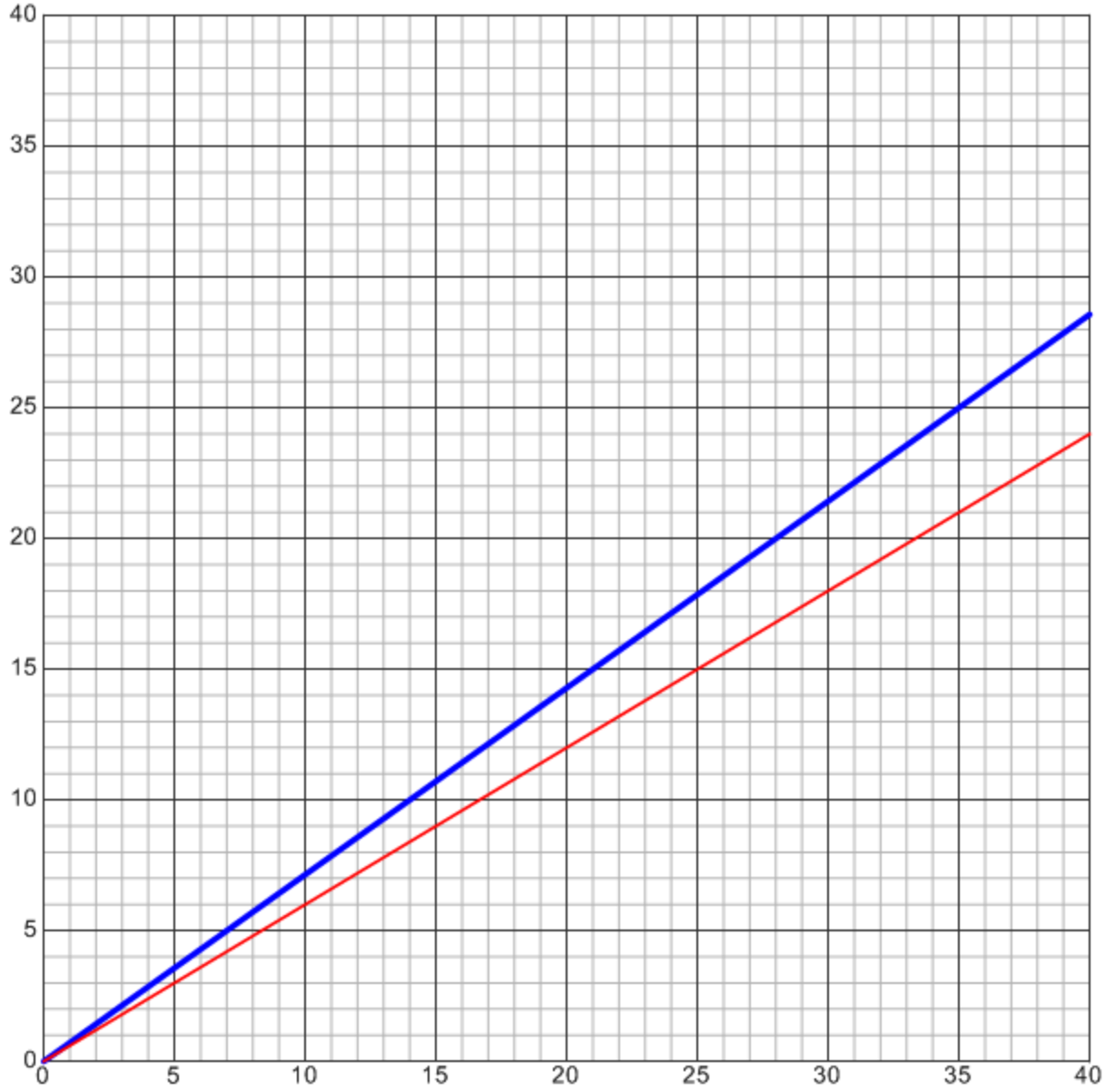
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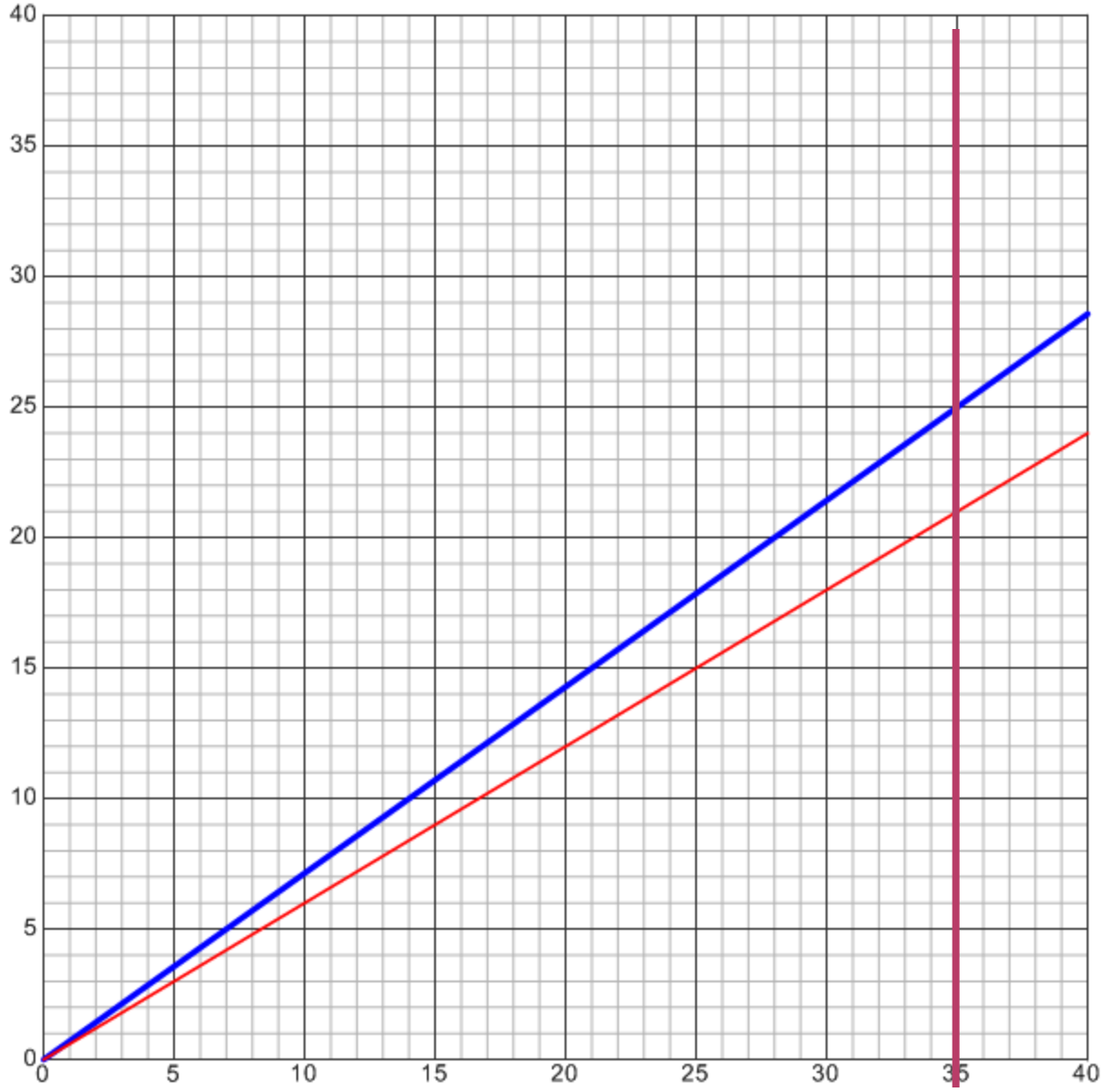
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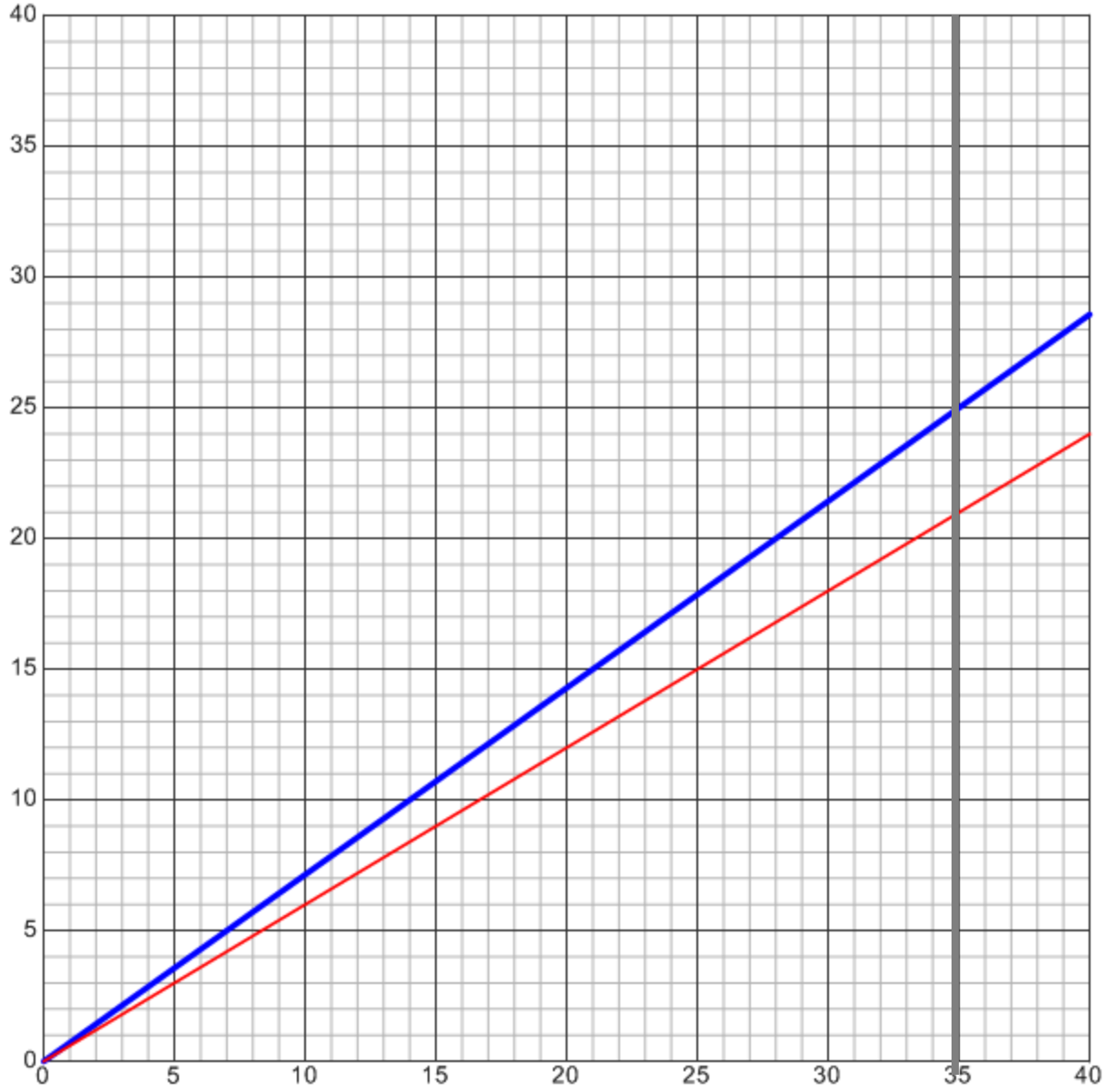
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OVERARCHING CONCERNS

- Teaching will not change in high schools and college math classes.
- The PARCC and Smarter Balanced Assessments will ultimately determine how successful the Common Core State Standards will be.
- The omissions and inappropriate location of certain concepts will not be addressed.

REFERENCES

- Using Classroom Assessment to Improve Student Learning. NCTM. 2011.
- NCTM Website. 2011.
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