



RECOGNIZING EFFECTIVE IMPLEMENTATION OF STANDARDS

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May 23, 2012

Effective Implementation demands the:

- Inclusion of on-going formative assessments;
- Inclusion of non routine tasks;
- Facilitation of mathematical discourse; and
- Opportunity to build new mathematical knowledge through problem solving.

Include Essential Components of Formative Assessment

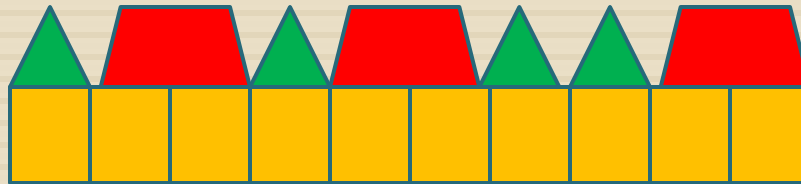
- Assess prior knowledge
- Use observational protocols
- Ask open questions
 - ▣ Engaging
 - ▣ Clarifying
 - ▣ Refocusing
- Collect exit cards/tickets to leave
- Listen twice as much as you talk

Pose Problems That Push Student Reasoning

- Problems that do not require an exact answer.
- Problems that peak a student's interest in the possible solutions.
- Problems that stimulate students to pose their own problems

Village Houses and Roofs

- In a small village houses are built in rows and either have a triangular roof or share a trapezoidal roof.



- How many possible roof arrangements are there if 10 houses are joined in a row?

Look for and make use of structure

- What pattern might you see in this problem?
- How do you know you have not miscounted?
- How do you know you have counted all the different ways in which the houses might look?

Use a conjecture board

- Pose a problem.
- Record student's responses as conjectures.
- Test the conjectures, revise them, test them again...and repeat the process.
- Conjectures remain on the board until a counter example is found or there is a negation.

Consider the following:

On a number line 7 is halfway
between 6 and 8

is

$\frac{1}{7}$ halfway between $\frac{1}{6}$ and $\frac{1}{8}$?

Construct viable arguments and critique the reasoning of others.

- How might you support your reasoning and “prove” that $\frac{1}{7}$ is, or is not, halfway between $\frac{1}{6}$ and $\frac{1}{8}$.
- Given a list of three consecutive unit fractions will the middle fraction ever be halfway between the first and last? How do you know?

Appropriate Responses to Student Ideas

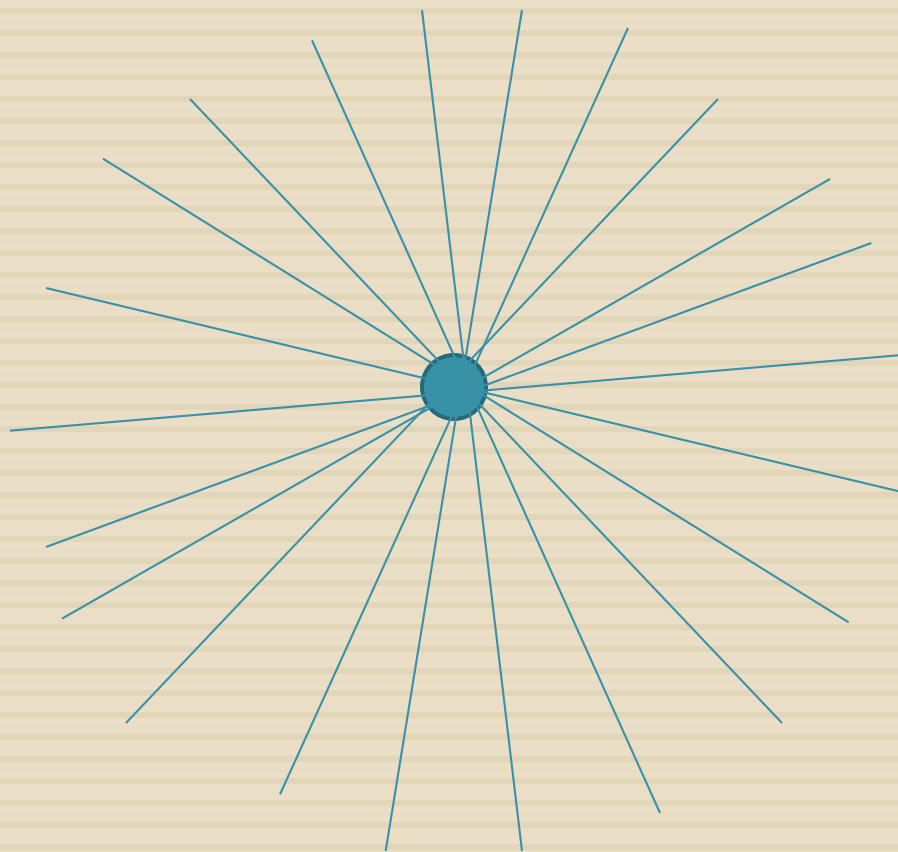
- React neutrally
- Record all responses on a conjecture board
- Listen for misconceptions
- Ask if anyone has a different idea or did it differently

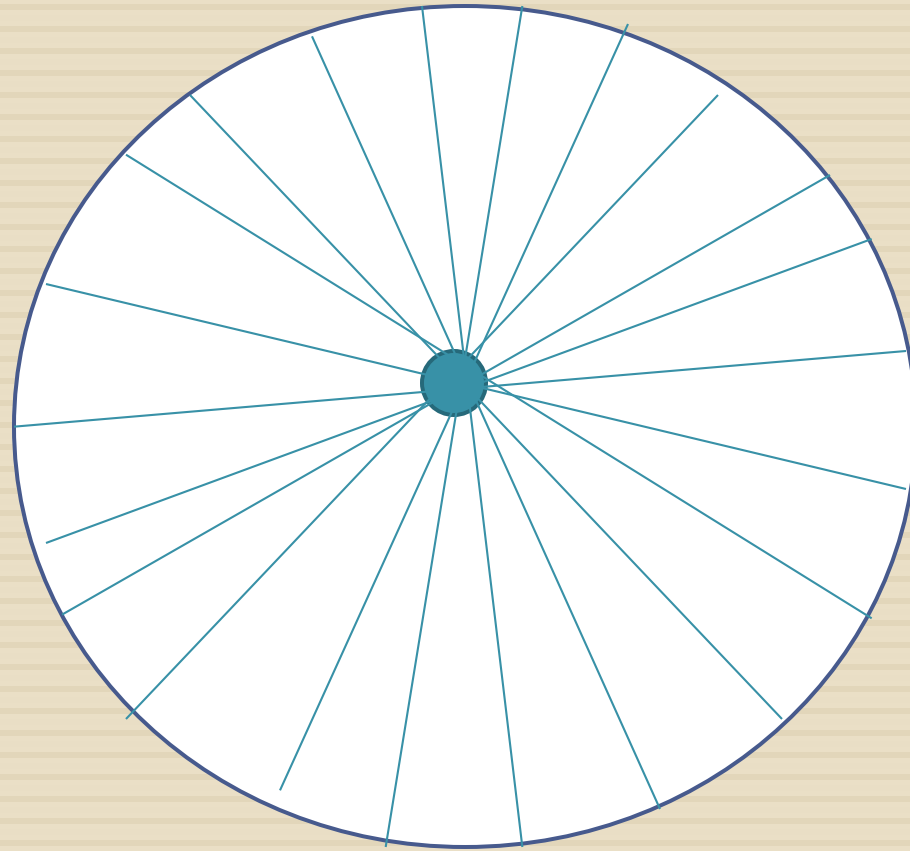
Range Question

- Tell me everything you know about a circle.

Respond With a Clarifying Activity

- Pass out a large post-it note or an index card and instruct students to put a point in the center.
- Next, draw as many line segments as possible through the point BUT be sure there is an equal length on both sides of the center point.





Role of the teacher

- Select and set up an interesting and rich mathematical task.
- Support students' exploration of the task
- Share work and discuss the task

What are the differences between the following two problems?

- $4/5$ is closer to 1 than $5/4$. Show why this is true on a number line.

- What is closer to 1?
 - a. $5/4$
 - b. $4/5$
 - c. $3/4$
 - d. $7/10$

Responsive Teaching

- Asks how you thought about the problem.
- Asks if that strategy will always work.
- Invites a student to explain why the strategy works or does not work.

Select and set up a mathematical task

- How does the task build on students' previous knowledge and experiences?
- What definitions, concepts, or ideas do students need to know to begin to work on the task?
- What questions will be asked to help students access their prior knowledge?
- What are all the ways the task can be solved?
- What particular challenges might the task present?

Reasoning and Sense-Making

Seamus ran 30 laps in 15 minutes.

Emma wrote $30 \div 15 = 2$.

What might the 2 represent?

Abby wrote $15 \div 30 = 0.5$.

What might the 0.5 represent?

Effective Teaching

- Puts the student first and foremost when planning lessons
- Asks questions that require more than a 1 sentence response
- Assesses where the student is on each given day
- Supports student learning, student discourse, student reasoning


Consider the following

Abby and Emma shared some beads in the ratio of 2 : 5. If Emma gave 30 beads to Abby, they would have the same number of beads. How many beads did Abby have in the beginning?

Without solving the problem...answer the following questions.

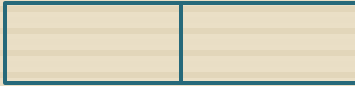
- How does the task build on students' previous knowledge and experiences?
- What definitions, concepts, or ideas do students need to know to begin to work on the task?
- What questions will be asked to help students access their prior knowledge?
- What are some the ways the task can be solved?
- What particular challenges might the task present?

What are some problem solving strategies?

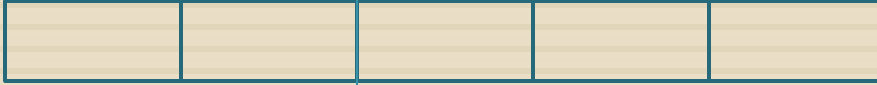


Starting Ratios

Abby

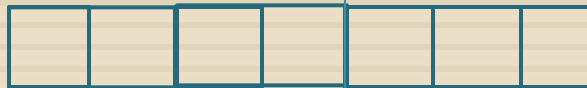


Emma

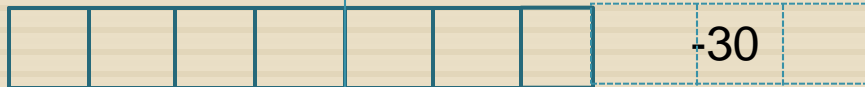


Resulting Ratios

Abby



Emma

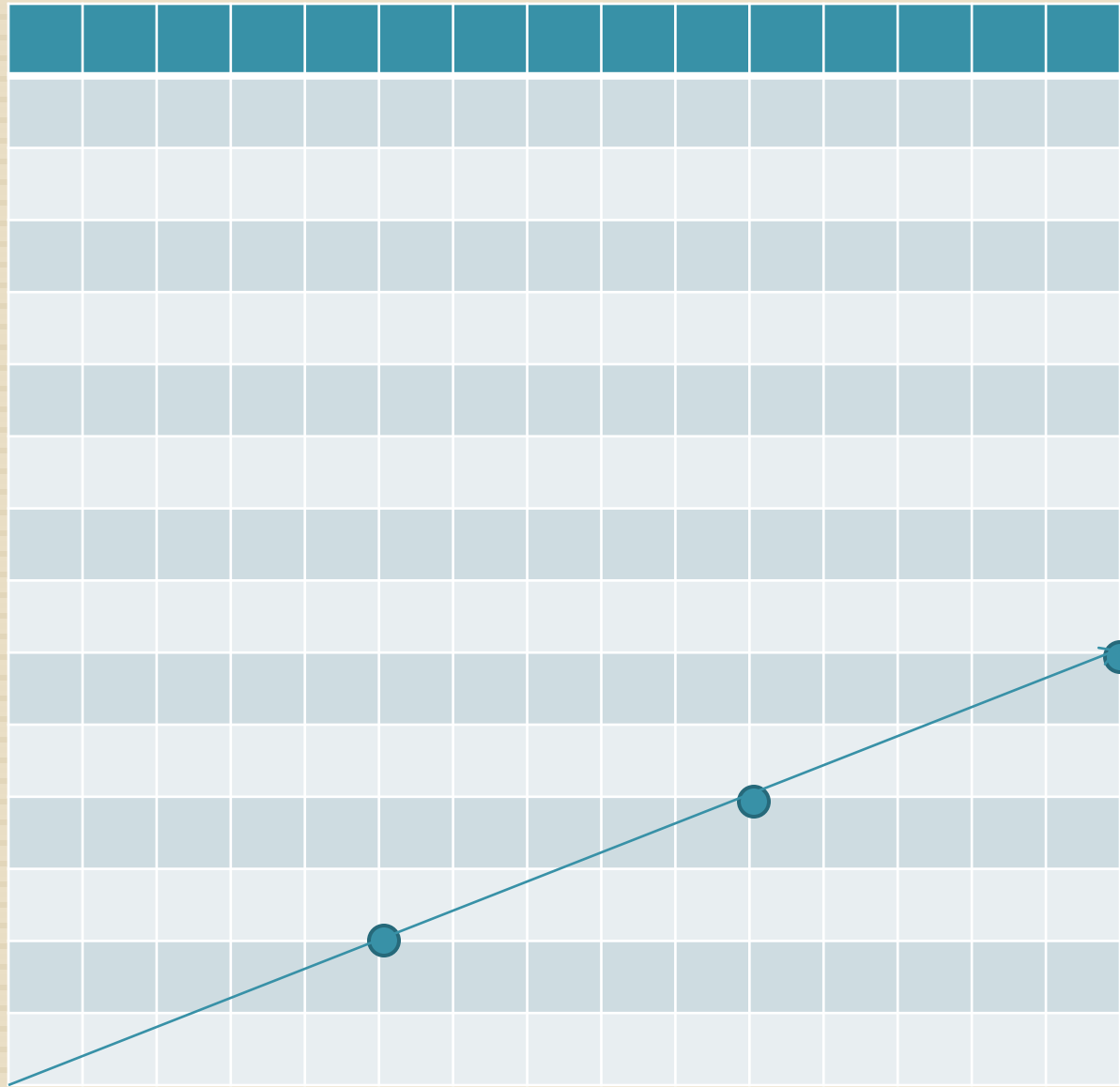


Algebraic Solution

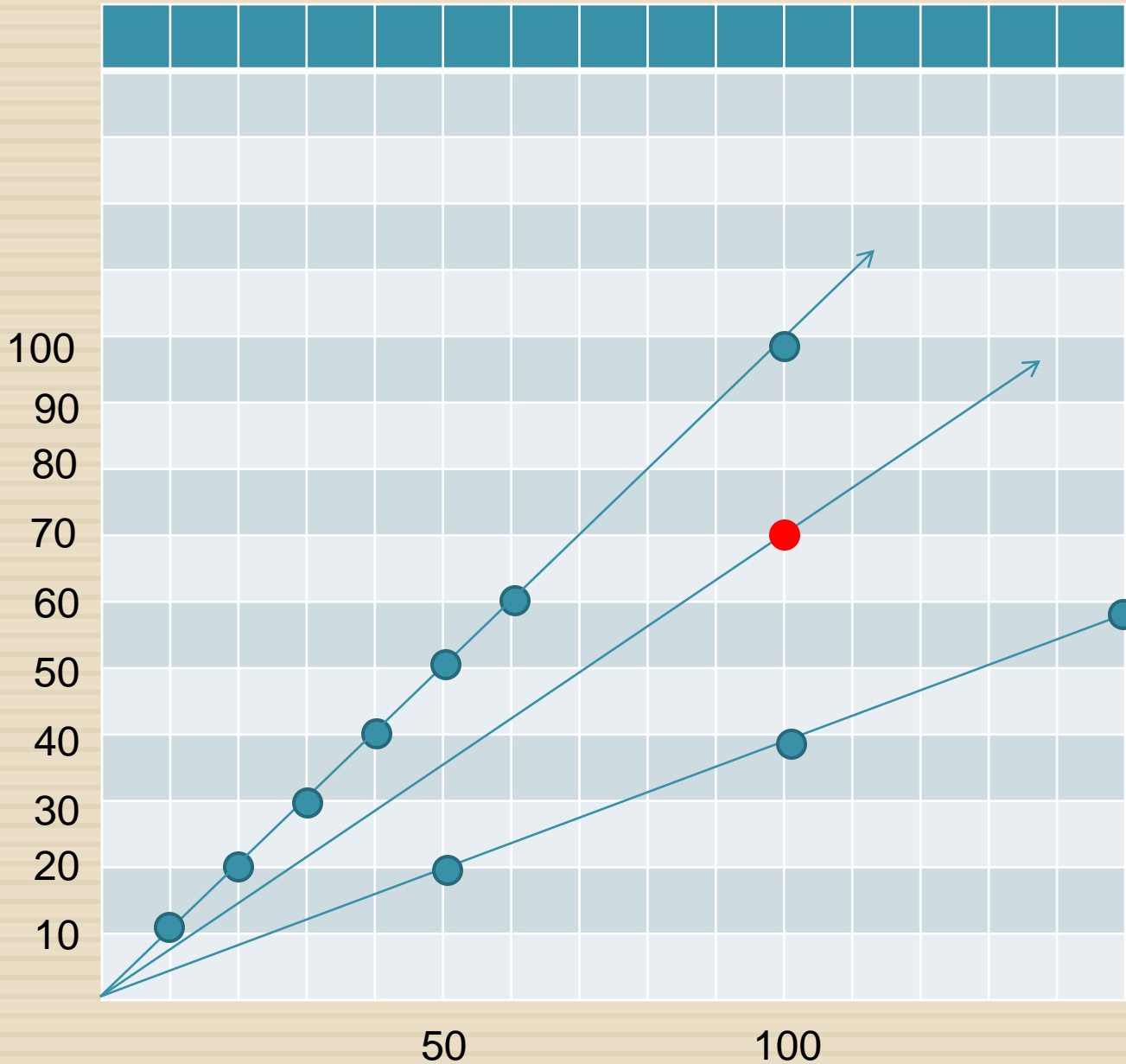
- $2x + 30 = 5x - 30$
- $60 = 3x$
- $x = 20$

Emma had 100 beads to begin with. Abby had 40.

Abby



Abby



Effective Teaching: Values Incorrect Answers

Wrong answers:

- Need to be part of the teaching/learning process
- Give information about what the student is thinking
- Might be an error of haste or an on-going misconception

Answer Getting Versus Learning Math

- United States
 - ▣ How can I teach my students to get the answer to the problem or computation?
- Japanese
 - ▣ How can I use this problem to teach the mathematics of this unit?

On a bulletin board in a high school

AREA

▶ $A = lw$

▶ $A = s^2$

▶ $A = 2\pi r^2$

Model With Mathematics

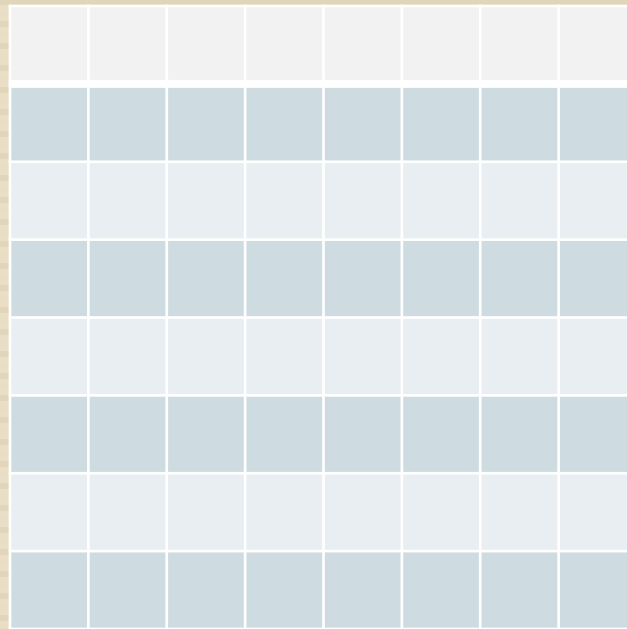
- Think about the mathematics content that is usually presented in an abstract manner.
- Think about square roots. What do they look like?
How might they be represented?
- What models can you use to make those abstract concepts approachable for all students?

Two Possible Models

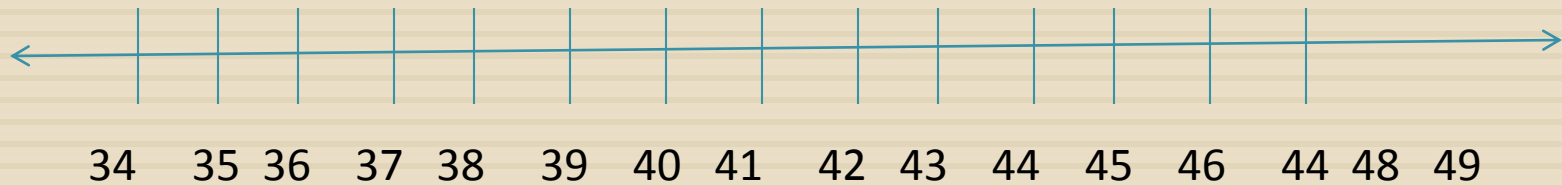
- Find a close approximation for the square root of 43 (without using a calculator)

Geometric Model

- On graph *paper* outline a 6 x 6 square.



Number Line Model



Use Multiple Representations

- A picture is worth a thousand words
- Which of the two models gives a visual image of what a square root looks like?
- Students learn best when they move from the concrete-pictorial-abstract.
- We jump too quickly to the abstract!

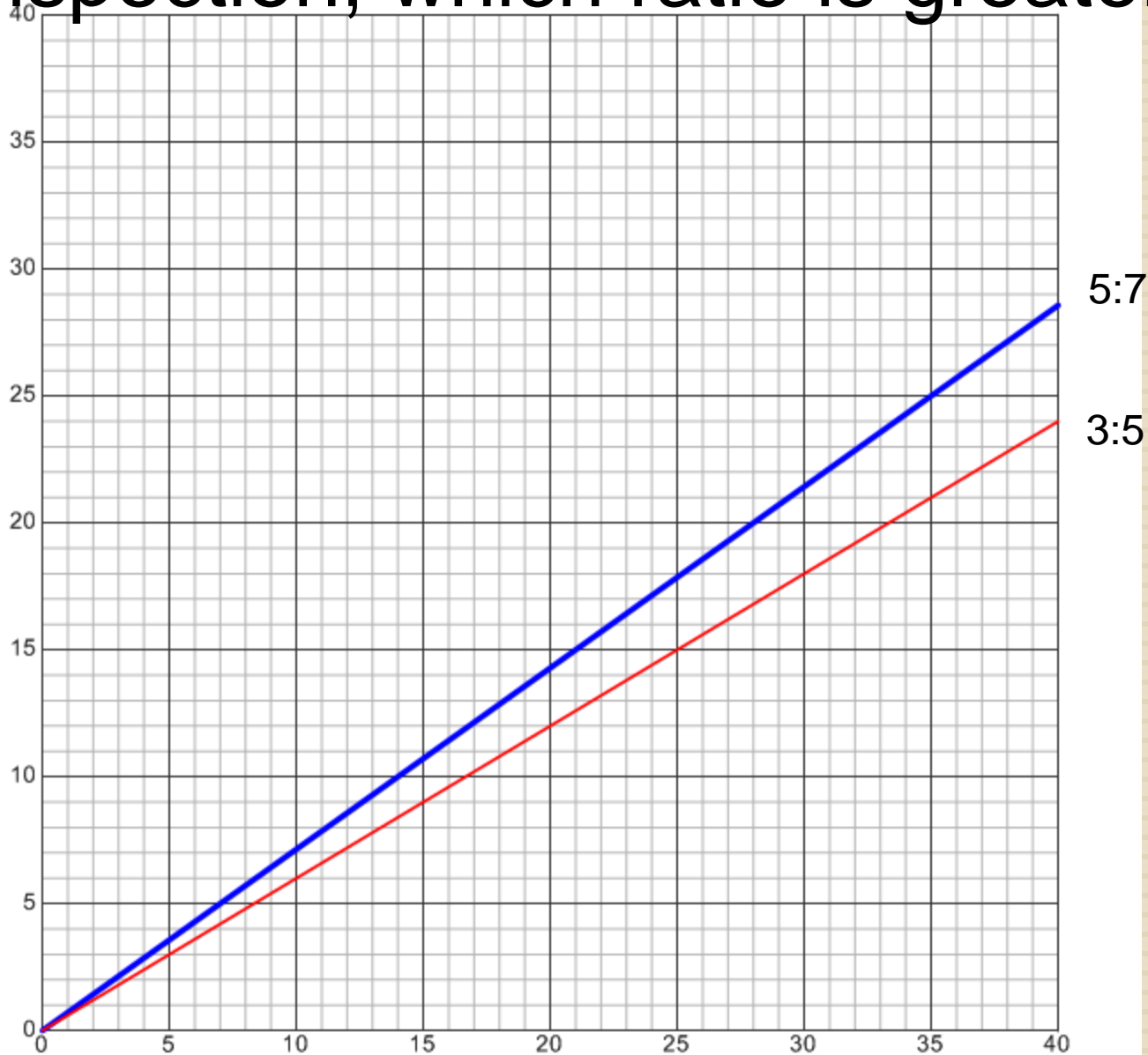
Fraction as Ratio

- Discrete
- Relationship between the numerator and the denominator
- Either a part to part or part to whole comparison
- Represent a Rate

Geometric Representations

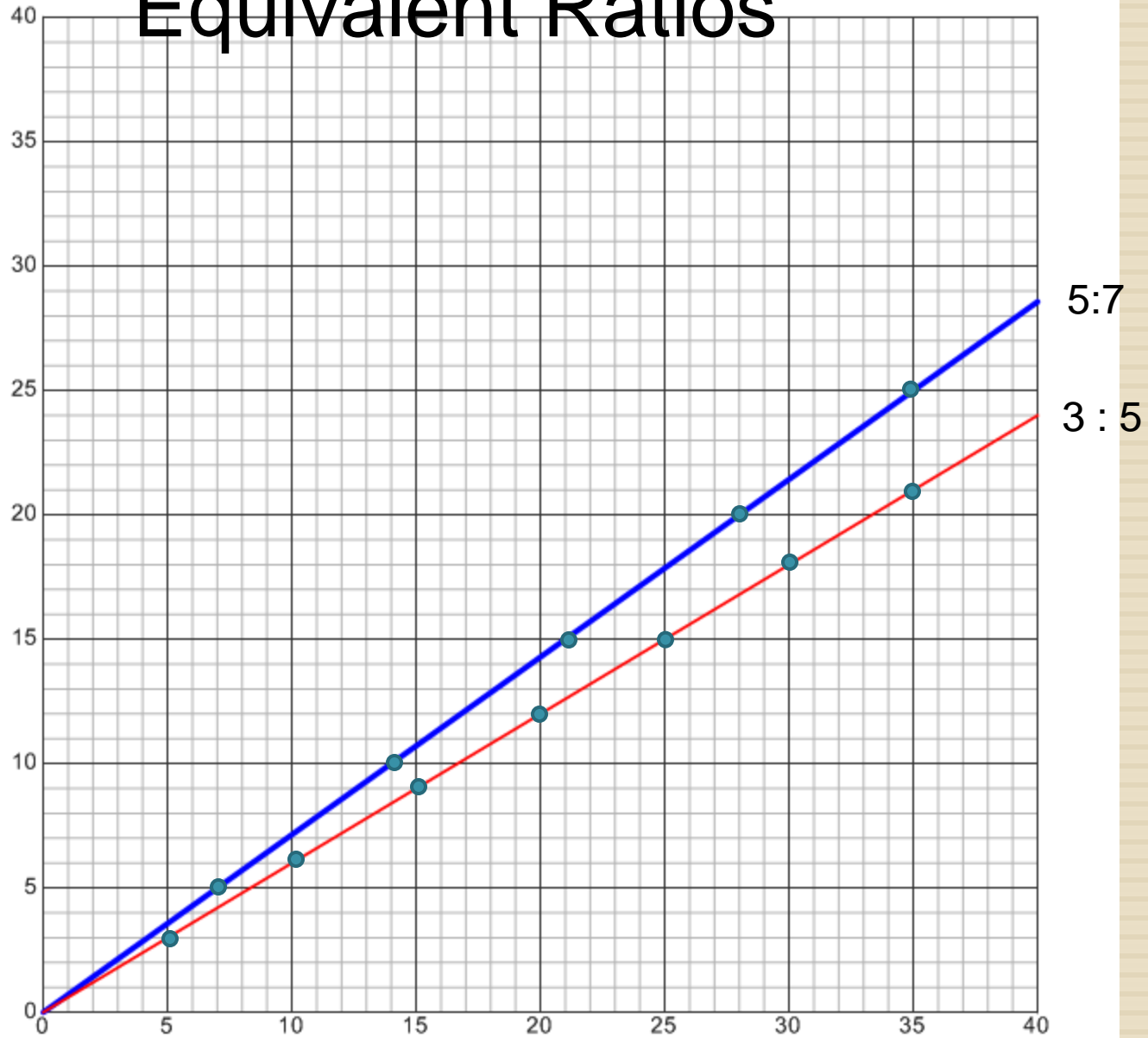
- What representations might you use to compare ratios?
- Which is greater $5:7$ or $3:5$? And why do you think I chose those two ratios?

By Inspection, which ratio is greater?

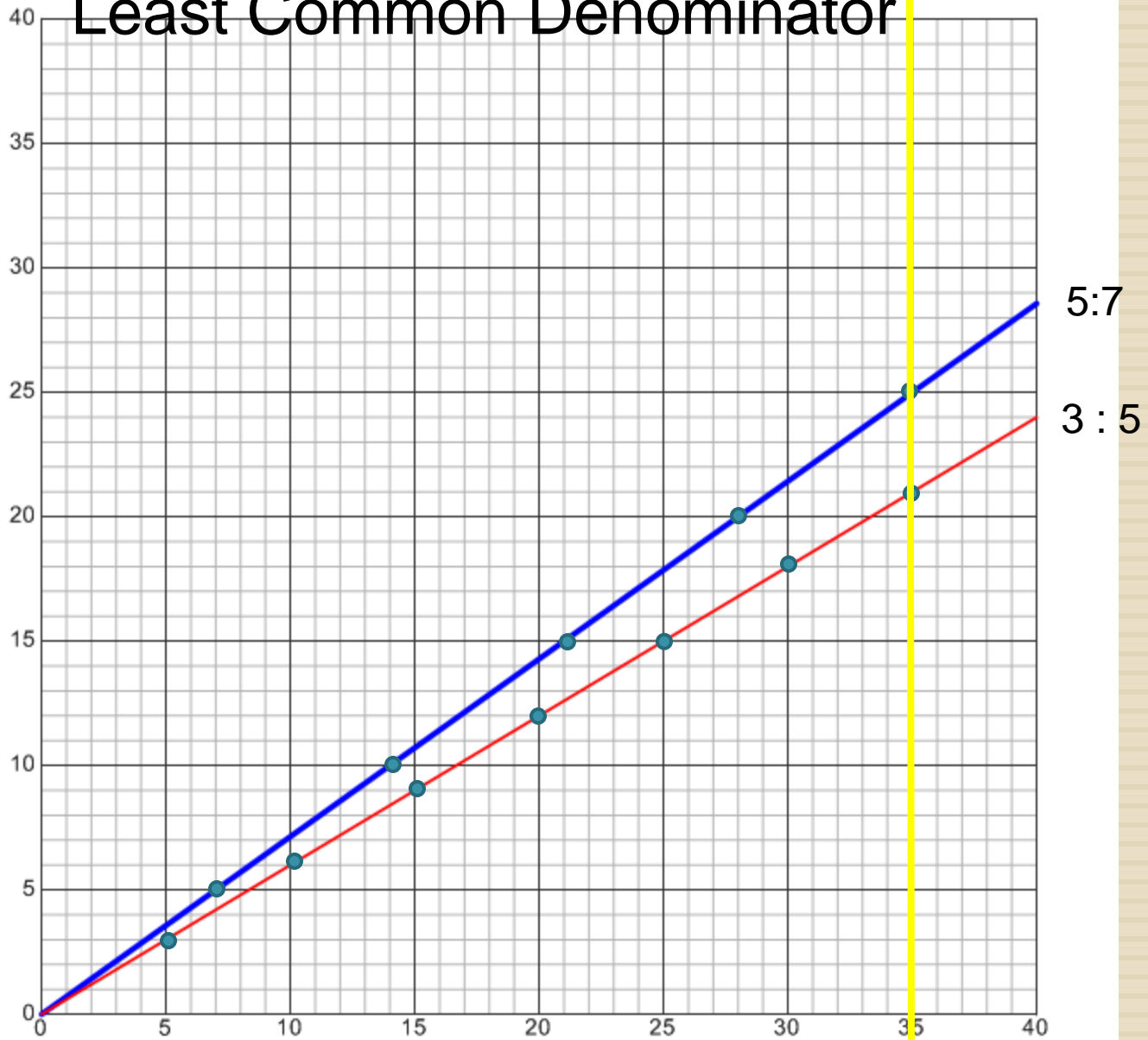


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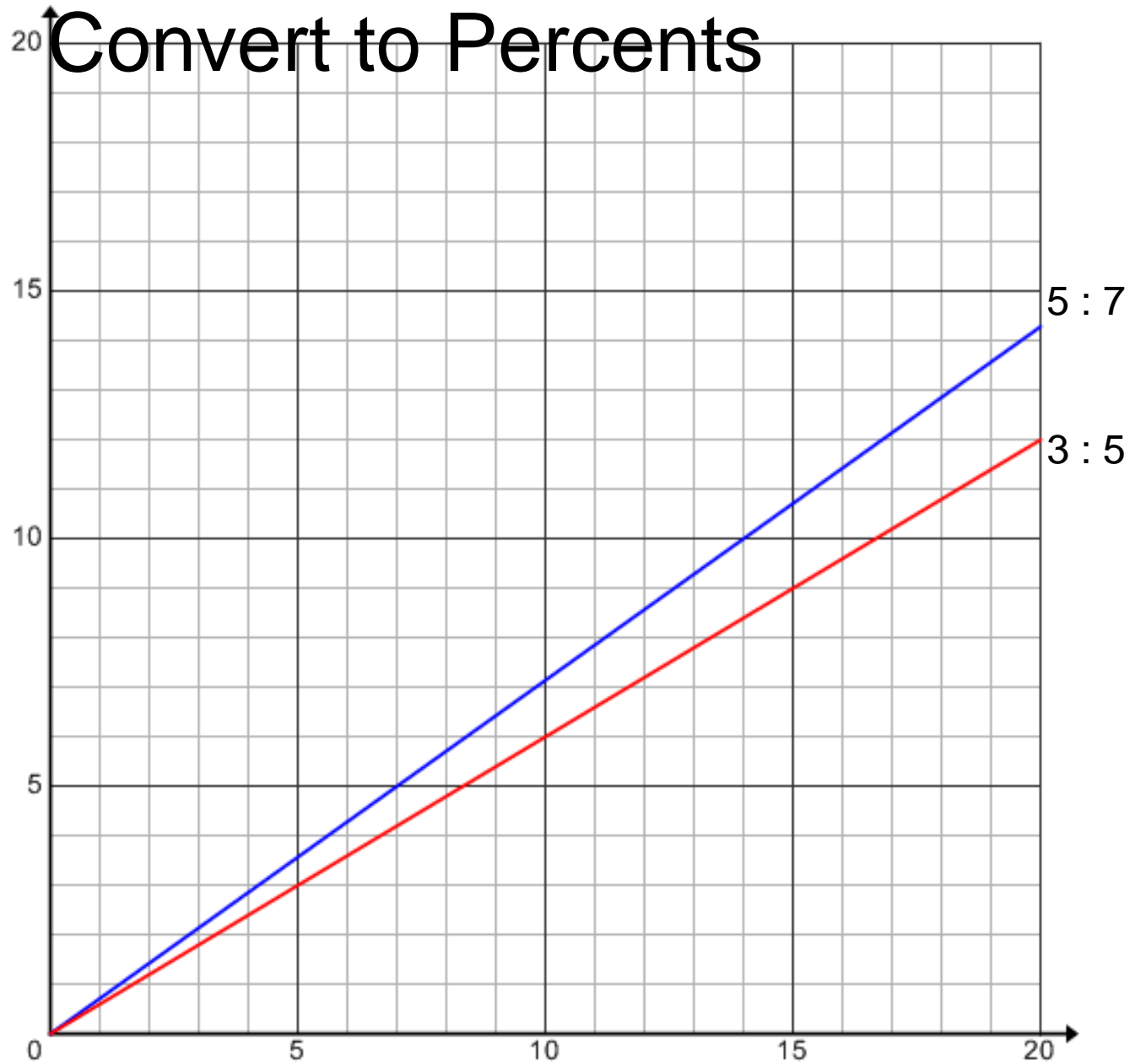
Equivalent Ratios



Least Common Denominator



Convert to Percents



I – We – You Teaching Strategy Is Ineffective!

- I- Teacher presents a topic, shows how to do the procedural steps.
- We- Students work through computations with the teacher
- You- Students practice the procedures with worksheets

Underlying Tenet of CCSS-M

- Teachers need to change the way in which they teach.
- If we keep doing what we have always done the results will continue to be the same...and that is not good enough.
- Coaches need to correct mathematical errors...even if it feels awkward.
- Administrators must ensure student ideas, misconceptions, and engagement are at the forefront of every classroom.