

Handouts for Matthews, M.E. (May 2012) "Recognizing effective implementation of Standards for Mathematical Practices: Task choice, classroom practices, and teacher questions." Massachusetts Mathematics Association of Teacher Educators Annual Symposium. MA.

Cognitive Demand of Tasks

Low-Level Cognitive Demands	High-Level Cognitive Demands
<p><i>Memorization Tasks</i></p> <ul style="list-style-type: none"> • Involve either producing previously learned facts, rules, formulae, or definitions <i>or</i> committing facts, rules, formulae, or definitions to memory. • Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • Are not ambiguous. Such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. • Have no connection to the concepts or meaning that underlay the facts, rules, formulae, or definitions being learned to reproduced. <p><i>Procedures Without Connections Tasks</i></p> <ul style="list-style-type: none"> • Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. • Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • Have no connection to the concepts or meaning that underlie the procedures being used. • Are focused on producing correct answers rather than on developing mathematical understanding. • Require no explanations or explanations that focus solely on describing the procedure that was used. 	<p><i>Procedures With Connections Tasks</i></p> <ul style="list-style-type: none"> • Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations help to develop meaning. • Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding. <p><i>Doing Mathematics Tasks</i></p> <ul style="list-style-type: none"> • Require complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). • Require students to explore and to understand the nature of mathematical concepts, processes, or relationships. • Demand self-monitoring or self-regulation of one's own cognitive processes. • Require students to access relevant knowledge in working through the task. • Require students to analyze the task and actively examining task constraints that may limit possible solutions strategies and solutions. • Require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

From Stein, M.K., Smith, M., Henningsen, M. & Silver, E.A. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development*. New York: Teachers College Press.

Teacher Questions

Question Type	Description	Examples
1. Gathering information, leading students through a method	Requires immediate answer Rehearses known facts/procedures Enables students to state facts/procedures	What is the value of x in this equation? How would you plot that point?
2. Inserting terminology	Once ideas are under discussion, enables correct mathematical language to be used to talk about them	What is this called? How would we write this correctly?
3. Exploring mathematical meaning and/or relationships	Points to underlying mathematical relationships and meanings. Makes links between mathematical ideas and representations	Where is this x on the diagram? What does probability mean?
4. Probing, getting students to explain their thinking	Asks students to articulate, elaborate or clarify ideas	How did you get 10? Can you explain your idea?
5. Generating Discussion	Solicits contributions from other members of class	Is there another opinion about this? What did you say, Justin?
6. Linking and applying	Points to relationships among mathematical ideas and mathematics and other areas of study/life	In what other situations could you apply this? Where else have we used this?
7. Extending thinking	Extends the situation under discussion to other situations where similar ideas may be used.	Would this work with other numbers?
8. Orienting and focusing	Help students to focus on key elements or aspects of the situation in order to enable problem-solving	What is the problem asking you? What is important about this?
9. Establishing context	Talks about issues outside of math in order to enable links to be made with mathematics	What is the lottery? How old do you have to be to play the lottery?

From Boaler, J. & Brodie, K. (2004). The Importance, nature, and impact of teacher questions. In McDougall, D.E & Ross, J. A. (Eds.). *Proceedings of the twenty-sixth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, Toronto: OISE/UT.

Connecting practices that help maintain cognitive demand with teacher questions

	Task builds on students' prior knowledge	High-level performance modeled	Sustained pressure for explanation and meaning	Scaffolding	Student self-monitoring	Teacher draws conceptual connections
1. Gathering information, leading students through a method						
2. Inserting terminology						
3. Exploring mathematical meaning and/or relationships						
4. Probing, getting students to explain their thinking						
5. Generating Discussion						
6. Linking and applying						
7. Extending thinking						
8. Orienting and focusing						
9. Establishing context						

Connecting teacher questions with CCSS Mathematical Practices

	2. Inserting terminology	3. Exploring mathematical meaning and/or relationships	4. Probing, getting students to explain their thinking	5. Generating Discussion	6-8. Linking, applying, extending, orienting, and focusing	9. Establishing context
Make sense of problems and persevere in solving them						
Reason abstractly and quantitatively						
Construct viable arguments and critique the reasoning of others						
Model with mathematics						
Use appropriate tools strategically						
Attend to precision						
Look for and make use of structure						
Look for and express regularity in repeated reasoning						